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Buckling Analysis of Laminated Composite Plates Using Finite Element Method

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Abstract

Laminated composite plate structures find numerous applications in aerospace, military and automotive industries. The role of transverse shear is very important in composites, as the material is weak in shear due to its low shear modulus compared to extensional rigidity. Hence, a true understanding of their structural behavior is required, such as the deflections, buckling loads and modal characteristics, the through thickness distributions of stresses and strains, the large deflection behavior and, of extreme importance for obtaining strong, reliable multi-layered structures. In this paper, buckling behavior of laminated composite plates under uniaxial and biaxial compression load using commercially available ANSYS software is studied. A number of finite element analyses have been carried out to study the effect of side-to-thickness ratios, aspect ratios, modulus ratios, ply orientation, and boundary conditions on nondimensionalized critical buckling load for three different materials. The numerical results showed, the effect of shear deformation is to decrease the nondimensionalized critical buckling load with the decrease of side to thickness ratio. The ANSYS results are validated with the results predicted by third order shear deformation theory.

Keywords: Laminated composite plate, nondimensionalized critical buckling load, Finite element method.

Introduction

Laminated composite materials are increasingly being used in a large variety of structures including aerospace, marine and civil infrastructure owing to the many advantages they offer: high strength/stiffness for lower weight, superior fatigue response characteristics, facility to vary fiber orientation, material and stacking pattern, resistance to electrochemical corrosion, and other superior material properties of composites. At the same time, the fabricated material poses new problems, such as failure due to delamination and pronounced transverse shear effects due to the high ratio of in-plane modulus to transverse shear modulus. Hence, a true understanding of their structural behavior is required, such as the deflections, buckling loads and modal characteristics, the through thickness distributions of stresses and strains, the large deflection behavior and, of extreme importance for obtaining strong, reliable multi-layered structures. The finite element method is especially versatile and efficient for the analysis of complex structural behaviour of the composite laminated structures.

In the past, the structural behavior of plates and shells using the finite element method has been studied by a variety of approaches. The initial theoretical research into elastic flexural-torsional buckling was preceded by Euler's (1759) treatise on column flexural buckling, which gave the first analytical method of

predicting the reduced strengths of slender columns, and by Saint-Venant's (1855) memoir on uniform torsion, which gave the first reliable description of the twisting response of members to torsion. However, it was not until 1899 that the first treatments were published of flexural-torsional buckling by Michel and Prandtl, who considered the lateral buckling of beams of narrow rectangular cross-section. Their work was extended by Timoshenko to include the effects of warping torsion in I-section beams. Most recently the invention of high-speed electronic computers exerted a considerable influence on the static and dynamic analysis of plates. Bryan (1891) gave the first solution for the problem by using the energy method to obtain the values of the critical loads. He assumed that the deflection surface of the buckled plate could be represented by a double Fourier series. Timoshenko (1925) used another method to solve the problem. He assumed that the plate buckled into several sinusoidal half waves in the direction of compression. When satisfying the boundary conditions, the equations formed a matrix problem which upon solving yields the critical load. The problem was discussed in many standard textbooks such as Timoshenko and Gere (1961) and Bulson (1970). Apart from simply supported plates, Timoshenko (1925) explored the buckling of uniformly compressed rectangular plates that are simply supported along two

opposite side's perpendicular to the direction of compression and having various edges along the other two sides. The various boundary conditions considered include *SSSS*, *CSCS*, *FSFS*, *FSCS*, *CSES* (*S* - simply supported edge, *F* - free edge, *C* - clamped or built-in edge and *E* - elastically restrained edge). The theoretical results were in good agreement with experimental results obtained by Bridget *et al.* (1934). The earliest accurate solution available is due to Levy (1942) for the case of *CCCC* plate with one direction uniaxial compression. He regarded the plate as simply supported, and then made the edge slopes equal to zero by a suitable distribution of edge-bending moments. Bleich (1952) obtained the critical load for the *ESES* plates with loaded edges elastically restrained. Schuette and McCulloch (1947) employed the Lagrangian multiplier to solve the buckling problem of *ESSS* plates. Walker (1967) used the Galerkin's method to give accurate values of critical load for a number of the edge conditions as mentioned before. He also studied the case of *ESFS* plates. Chen and Bert (1976) investigated optimal design of simply supported rectangular plates laminated to composite material and subjected to uniaxial compressive loading. Numerical results are presented for optimal-design of plates laminated of glass/epoxy, boron/epoxy, and carbon/epoxy composite materials. Leissa gave a summary of the buckling and post-buckling studies of composite laminated plates up to 1986, and then he reviewed the development of buckling analysis of laminated composite plates with linear effective constitutive properties. Later a more detailed account of the research on the buckling and postbuckling before 1995 was presented by Noor. A.K. Shrivastava & R.K. Singh (1998) studied the effect of aspect ratio on buckling behavior. In this paper an attempt has been made to study the effect of aspect ratio, d/b & d/D on the buckling of laminated composite plates by FEA using ANSYS. Radu and Chattopadhyay (2000) used a refined higher order shear deformation theory to investigate the dynamic instability associated with composite plates with delimitation that are subject to dynamic compressive loads. The procedure is implemented using the finite element method. Hwang and Mao (2001) conducted the non-linear buckling and post-buckling analyses to predict the delamination buckling load and elimination growth load. A procedure for determining the buckling load of the aluminum rectangular plate is presented by Supasak and Singhatanadgid (2002). Buckling load of aluminum rectangular plates are determined using four different techniques, i.e. (1) a plot of applied load vs. out-of-plane displacement, (2) a plot of applied load vs. end shortening, (3) a plot of applied load vs. average in-plane strain, and (4) the Southwell plot. In this study, buckling loads determined from different experiment methods

were compared with the theoretical buckling loads. Wang and Lu (2003) were carried out an investigation to understand the buckling behavior of local delaminating near the surface of fiber reinforced laminated plates under mechanical and thermal loads. Xiang *et al.* (2003) used the Ritz method to solve the buckling problem of rectangular plates with an internal line hinge under both uniaxial and biaxial loads. The buckling factors are generated for rectangular plates of various aspect ratios, hinge locations and support condition Shukla and Kreuzer (2005) proposed a formulation based on the first-order shear deformation theory and von-Karman-type nonlinearity to estimate the critical/buckling loads of laminated composite rectangular plates under in-plane uniaxial and biaxial loadings. Different combinations of simply supported, clamped and free boundary conditions were considered. The effects of plate aspect ratio, lamination scheme, number of layers and material properties on the critical loads were studied. Pannok and Singhatanadgid (2006) studied the buckling behavior of rectangular and skew thin composite plates with various boundary conditions using the Ritz method along with the proposed out-of-plane displacement functions. Buket Okutan Baba (2007) studied the influence of boundary conditions on the buckling load for rectangular plates. Numerical and experimental studies are conducted to investigate the effect of boundary conditions, length/thickness ratio, and ply orientation on the buckling behavior of E-glass/epoxy composite plates under in-plane compression load. Buckling analysis of the laminated composites is performed by using finite element analysis software ANSYS. Pein and Zahari (2007) studied the structural behavior of woven fabric composites subject to compressive load which is locking. A parametric study is performed to investigate the effect of varying the fiber orientations and different central hole sizes onto the strength of the laminates. Murat Yazici (2008) studied the influence of square cut-out upon the buckling stability of multilayered, steel woven fiber-reinforced polypropylene thermoplastic matrix composite plates are studied by using numerical and experimental methods. Y.X. Zhang, C.H. Yang / Composite Structures (2009) studied the plates subjected to mechanical loads, while the investigations on the post buckling response of composite plates subjected to thermal or combined thermal and mechanical loadings are rather limited. Considerable efforts have been made for the numerical analysis of the buckling and post buckling analysis over the years. Nagendra singh gira and nagendra kumar gira (2010) studied the Buckling load factors have been determined for different aspect ratio, d/b ratio & d/D ratio.. Hu investigated the influence of in-plane shear nonlinearity on buckling and post buckling responses of composite plates under uniaxial compression and bi-axial

compression and of shells under end compression and hydrostatic compression. They also investigated the nonlinear buckling of simply-supported composite plates under uniaxial compression, and of composite laminate skew plates under uniaxial compressive loads. Hurang Huetal (2011) investigated the buckling behavior of a graphite/epoxy symmetrically laminated composite rectangular plate under parabolic variation of axial loads. The influence of plate aspect ratio and fiber orientation has been investigated. Jana and Bhaskar carried out a buckling analysis of a simply supported rectangular plate subjected to various types of non-uniform compressive loads. Priyanka dhurvey and Nd Mittal (2012) investigated the buckling behavior of orthotropic laminate using finite element analysis. The effect of fibre orientation on buckling behavior in a rectangular composite laminate with central circular hole under uniform in-plane loading has been studied by using finite element method.

From the literature, it is evident that most of the studies are based on the numerical approach, experimental and analysis. The literature review it was found that most of the studies were focused on unidirectional. Recently finite element analysis has been received considerable alter from to analyse the laminates composite plates. The main objective of this study deals with the buckling behavior of laminated composite plates subjected to uniaxial and biaxial compressive load using a finite element analysis. The buckling analysis is investigated for various side to thickness ratios (a/h), and aspect ratios (a/b), modulus ratio (E_1/E_2), different boundary conditions, and different fiber orientations for different materials.

Geometry of Shell Element

There are many element types, in ANSYS software, available to model layered composite materials. In our FE analysis, the shell91 structural shell element is used. It is designed to model thin to moderately thick plate and shell structures with a side-to-thickness ratio of roughly 10 or greater. The shell91 structural shell element allows up to 16 uniform-thickness layers. Alternatively, the sandwich option is turned of the element allows 100 thick layers with thicknesses that may vary bilinearly over the area of the layer. It also has an option to offset the nodes to the top or bottom surface. The “shell 91” element is more efficient than “shell99”. Each layer of the laminated shell element may have a variable thickness (TK). The thickness is assumed to vary bilinearly over the area of the layer, with the thickness input at the corner node locations. If a layer has a constant thickness, only TK(I) need be input. If the thickness is not constant, all four corner thicknesses must be input using positive values. With nonlinear material

properties, the thickness of any one layer may not exceed one-third of the total thickness of the element. The total thickness of each shell element *must* be less than twice the radius of curvature, and *should* be less than one-fifth the radius of curvature. The geometry of the non linear layered structural shell element is shown in Fig 1.

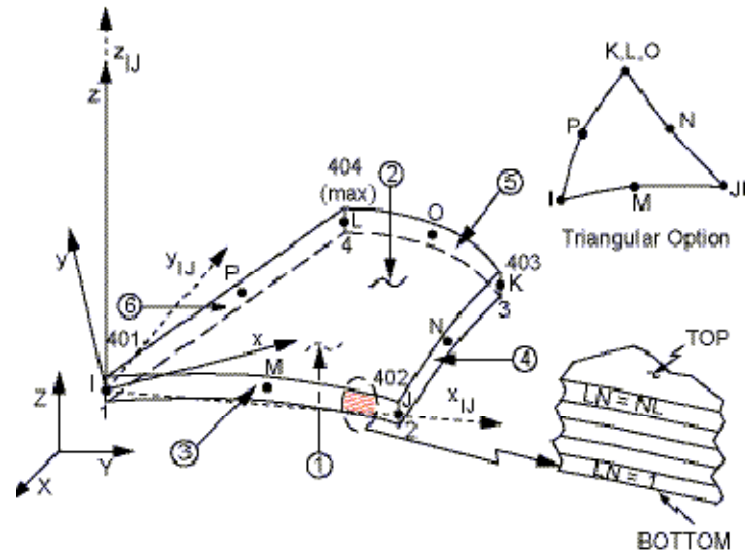


Figure 1: Geometry of 8-node element with six degrees of freedom

Finite Element Analysis

Finite element analysis includes three steps. (a)Preprocessing (b) analysis (c) post processing. Preprocessing includes modeling of the plate and applying boundary conditions like constraints, symmetry conditions, and loads. The following properties of material are as fallows.

Material properties (Gpa)

Material	E ₁	E ₂	E ₃	G ₁₂	G ₂₃	G ₁₃	V ₁₂	V ₂₃	V ₁₃
Boron/epoxy	206.9	20.7	20.7	5.2	5.2	5.2	0.3	0.3	0.3
Graphite/epoxy	155	12.1	12.1	0.248	0.458	0.248	4.4	3.2	4.4
Carbon/epoxy	206.9	5.2	5.2	2.6	2.6	2.6	0.25	0.25	0.25

To create model first area is created. Then the plate is meshed. Then load is applied on the plate. The plate is subjected to different boundary conditions. The all edges of the plate is constrained by all degrees of freedom and to the right end a buckling load of 1 N is applied. Unit loads are usually sufficient (that is, actual load values need not be specified). The Eigen values calculated by the buckling analysis represent critical buckling load. Therefore, if a unit load is specified, the load factors represent the buckling loads. Here analysis is done in two stages. In the first stage static analysis is done and Pre stress effects [PSTRES] must be activated. Eigen value buckling analysis requires the stress stiffness matrix to be calculated. In the second stage Eigen

buckling analysis done. After solving the problem the mode shapes and normal stress distribution is observed in the postprocessor. The output from the solution mainly consists of the Eigen values, which are printed as part of the printed output. The Eigen values represent the critical buckling load.

4. RESULTS AND DISCUSSIONS

The results show an excellent correlation to the results given by Reddy (2003). J.N Reddy results are validated with ANSYS software accurate results are obtained based on these results to follow the same procedure find out the maximum buckling load by increasing number of layers.

Table.1: Non-dimensional uniaxial buckling

loads, $\bar{N} = N_{cr} \times \frac{a^2}{E_2 \times h^3}$ of simply supported symmetric (0/90/0/0) cross ply laminates.

a/h	TSDT	ANSYS	%Error
5	11.997	11.46	4.47611903
10	23.34	23.223	0.50128535
20	31.66	31.508	0.47757423
50	35.347	35.257	0.2546185
100	35.953	35.904	0.13628904

The composite plate with constant thickness is analyzed by varying the number of layers up to 16 to select the optimum number of plies. In this case, the maximum critical buckling load is obtained at 7 layers for uniaxial compressive load. The effect of aspect ratio, side to thickness ratio, modulus ratio, boundary conditions, fiber orientations on the maximum critical buckling load for three different materials at optimum number of layers has been investigated.

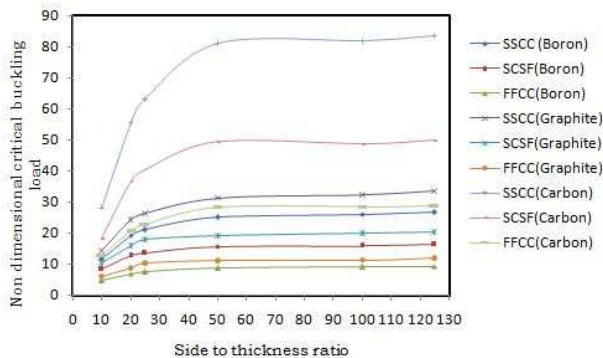


Fig.2: Effect of side to thickness ratios (a/h) on Non-dimensional critical buckling load (\bar{N}) (under uniaxial compression) for stacking sequence (45/-45/45/-45/45/-45/45)

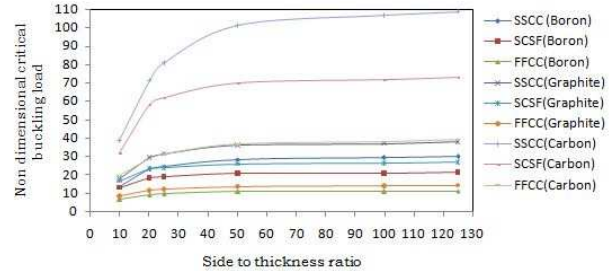


Fig.3. Effect of side to thickness ratios (a/h) on dimensional critical buckling load (\bar{N}) (under uniaxial compression) for stacking sequence (0/90/0/90/0/90/0)

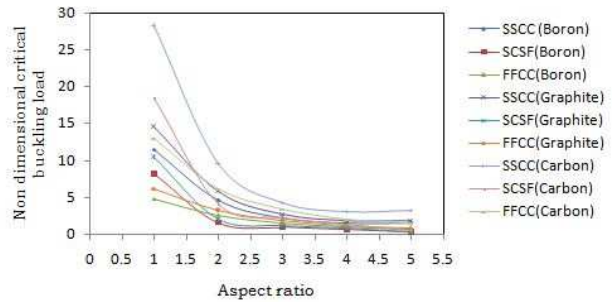


Fig .4. Effect of Aspect ratios (a/b) on Non dimensional critical buckling load (\bar{N}) (under uniaxial compression) for stacking sequence (45/-45/45/-45/45/-45/45)

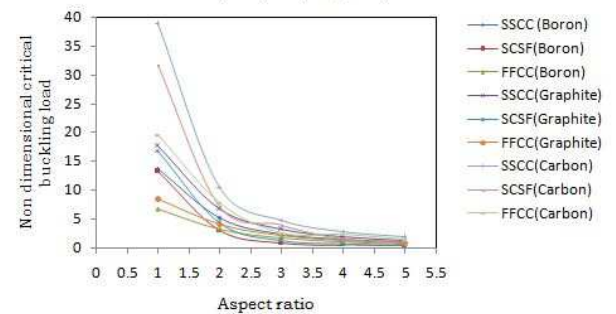


Fig.4. Effect of Aspect ratios (a/b) on Non-dimensional critical buckling load (\bar{N}) (under uniaxial compression) for stacking sequence (0/90/0/90/0/90/0)

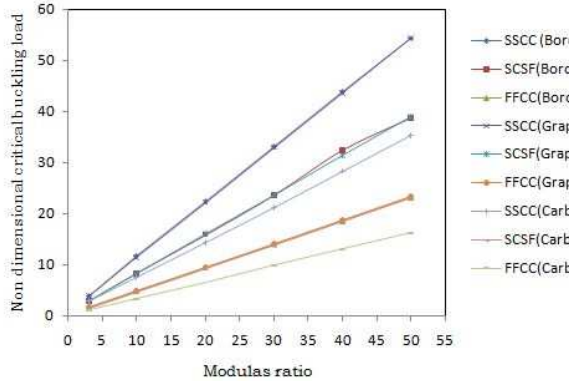


Fig. 6. Effect of modulus ratio (E_1/E_2) on Non-dimensional critical buckling load (\bar{N}) (under uniaxial compression) for stacking sequence (45/-45/45/-45/45)

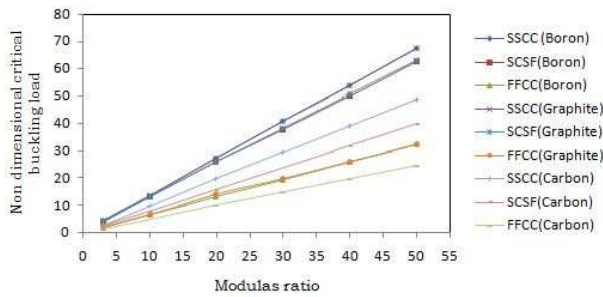


Fig.7. Effect of modulus ratio (E_1/E_2) on Non-dimensional critical buckling load (\bar{N}) (under uniaxial compression) for a stacking sequence (0/90/0/90/0/90/0)

Fig. 2 and Fig.3 represent the variation of non-dimensional critical buckling load against side-to-thickness ratio under uniaxial compression. From figures it is important to observe that, the effect of shear deformation is significant for $a/h \leq 50$ and beyond 50 the shear deformation is diminishes or decreases. In this study side-to-thickness ratio was changed from 10 to 125. Also observed that, increase of side-to-thickness ratio increases the non-dimensional critical buckling load. The carbon/epoxy material showed the highest non-dimensional critical buckling load under SSCC condition among those materials. The difference in critical buckling load between SSCC (carbon) and SCSF (carbon) is 32.77% and SSCC (carbon) and FFCC (carbon) is 64% at a/h is 120. However, the maximum critical buckling load is observed in stacking sequence [0/90/0/90/0/90/0].

Fig. 4 and Fig. 5 represent the variation of non-dimensional critical buckling load against Aspect ratio under uniaxial compression. From the graphs it is observed that the non-dimensional critical buckling load is maximum at lower values of aspect ratio and decreases with the increase of aspect ratio. This is due to the

decrease of bending-extensional twisting (D) stiffness with increase of aspect ratio. The effect of bending-extensional twisting (D) stiffness is quite significant for aspect ratio less than 3.5. In this study, the aspect ratio was changed from 1 to 5. For an aspect ratio of 1, showed the highest critical buckling load. Also it is observed that the non-dimensional buckling load decreases with increase in aspect ratio. The carbon/epoxy material showed the highest non-dimensional critical buckling load under SSCC condition among those materials. The difference between in critical buckling load between SSCC (carbon) and SCSF (carbon) is 18.1% and SSCC (carbon) and FFCC (carbon) is 49.21% at a/b is 1. However, the maximum critical buckling load is observed in stacking sequence [0/90/0/90/0/90/0].

Fig. 6 and Fig 7 represent the variation of non-dimensional critical buckling load against modulus ratio under uniaxial compression. From the figures it is observed that, the non dimensional buckling load increases with increases of modulus ratio. This is due the increase of modulus of elasticity. In this study modulus ratio was changed from 3 to 50. For modulus ratio of 50, showed the highest critical buckling load. Also it is observed that the non-dimensional critical buckling load increases with increase of modulus ratio. The Graphite/epoxy material showed the highest non-dimensional critical buckling load under SSCC condition among those materials. The difference between in critical buckling load between SSCC (Graphite) and SCSF (Graphite) is 21.9% and SSCC (carbon) and FFCC (carbon) is 55.37% at E_1/E_2 is 50. However, the maximum critical buckling load observed in stacking sequence [15/-15/15/-15/15/-15/15].

Conclusion

The buckling analysis of composite laminated plates under uniaxial compression using finite element analysis has been studied. The effect of aspect ratio, side to thickness ratio, modulus ratio, boundary conditions, and fiber orientations on the maximum critical buckling load for three different materials at optimum number of layers has been investigated. From the results; it was observed that, the non dimensional critical buckling load increases with increase of side to thickness ratio and modulus ratio under uniaxial compression due to the effect of shear deformation. As the aspect ratio increases, the effect of bending-extensional twisting stiffness is to decrease the critical buckling load under uniaxial compression.

References

[1] A K Sreevastva, R.K Singh. Effect of aspect ratio on buckling of composite plates-.Journal of

- Composites Science and Technology 1 (1999) 439-445.
- [2] V. Piscopo, "Refined Buckling Analysis of Rectangular Plates under Uniaxial and Biaxial Compression", World Academy of Science, Engineering and Technology. Vol. 70, 2010, pages 555-562.
- [3] Buket Okutan Baba and Aysun Baltaci. Buckling characteristics of symmetrically and anti-symmetrically laminated composite plates with central cutout, -Applied Composite Materials – 14(2007):265–276
- [4] Chainarin Pannok and Paired Singhatanadgid. Buckling analysis of composite laminate rectangular and skew plates with various edge support conditions. -The 20th Conference of Mechanical Engineering Network of Thailand (2006) 18-20.
- [5] K.K. Shukla; Y. Nath; E. Kreuzer, and K.V. Sateesh Kumar. Buckling of Laminated Composite Rectangular Plate, Journal of aerospace engineering, vol. 18(2005):215.
- [6] K. M. Jeong and H. G. Beom, Buckling Analysis of an Orthotropic Layer Bonded to a Substrate with an Interface Crack, Journal of composite materials, Vol. 37, (2003):522
- [7] ANSYS, *Theory Reference Manual*, and ANSYS *Element Reference*, (2010).
- [8] Hwang and Ching-Ping Mao, Failure of Delaminated Carbon/Epoxy Composite Plates under Compression- Composites Science and Technology (2001), 1513–1527 24.
- [9] Timothy L. C. Chen and Charles W. Bert. Design of composite-material plates for maximum uniaxial compressive buckling load - Proc. Okla. Acad. Sci. (1976): 104-107.
- [10] X. Wang and G. Lu . Local buckling of composite laminar plates with various delaminated shapes – *Journal of Thin-Walled Structures* (2003) 493–506.
- [11] Krishnamurthy C.S., "Finite Element Analysis: Theory and Programming", Tata McGraw-Hill Publishing Company Limited 1994.
- [12] Reddy J.N., "An Introduction to the Finite Element Method", Tata McGraw-Hill Publishing Company Limited, III edition 2006.
- [13] Reddy J.N., "Mechanics of laminated composites plates and shells" II edition CRC press (2003).
- [14] Jia Xie, Qing-Qing Ni *, Masaharu Iwamoto, "Buckling analysis of laminated composite plates with internal supports" *Composite Structures* 69 (2005) 201–208.
- [15] Lakshminarayana, R. Vijaya Kumar, "Buckling analysis of quasi-isotropic symmetrically laminated rectangular composite plates with an elliptical/circular cutout subjected to linearly varying in-plane loading using fem" *International journal mechanics*.
- [16] Zhang .Y.X, Yang.C.H, Recent developments in finite element analysis for laminated composite plates 2009; *Compos Struct*; 88; 147-157.
- [17] P. Jana, K. bhaskar; Stability analysis of simply-supported rectangular plates under non-uniform uniaxial compression using rigorous and approximate plane stress solutions. *Thin-Walled structures* 2006; 44; 507-516.
- [18] Jae-Hoon Kang, Arthr W. Leissa, Exact solutions for the buckling of rectangular plates having linearly varying in-plane loading on two opposite simply supported edges.
- [19] Tinh Quoc Bui, "Buckling analysis of simply supported composite laminates subjected to an in-plane compression load by a novel mesh-free method" *Vietnam Journal of Mechanics*, VAST, Vol. 33, No. 2 (2011), pp. 65 – 78.
- [20] Priyanka Dhurvey and N D Mittal, "Buckling Behavior of an Orthotropic Composite Laminate using Finite Element Analysis" *International Journal of Scientific Engineering* (2012) and *Technology* (ISSN : 2277-1581) www.ijset.com, Volume No.1, Issue No.4, pg : 93-95